An EOQ Model with Random Yields for Investing to Yield Variability under a Limited Capital Budget

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Abstract

In this paper, we extend our previous work to consider the determination of lot sizing and investments in the reduction of setup cost and of yield's standard deviation for EOQ model with random yields and a limited capital budget. The setup cost and yield's standard deviation are assumed as the functions of capital expenditure, respectively. In addition, we show that the total relevant cost function is convex and develop a solution procedure to determine the optimal lot size and the capital investments with a limited capital budget to minimize the expected total annual cost. Finally, a numerical example is presented to illustrate the procedure and to delineate the relationships between the optimal lot size and these investment decisions. These results evidently show that the performance on costs savings can be improved significantly through capital investments. Managerial implications are also included.

Keywords: Production; Lot sizing; Random yields; Investment decisions

1. Introduction

In recent years, random yields lot sizing problem has become an important research topic in production and inventory area. Several papers have reported the implications of yield randomness on lot sizing decisions. For an extensive review on many other lot sizing problems with random yields, interested readers are referred to the excellent paper by Yano and Lee[1]. In particular, Silver[2] was one of the earliest authors that extended the classical economic order quantity (EOQ) model to include the case where the quantity received from the supplier does not necessarily match the quantity requisitioned. Following Silver[2], Parlar and Berkin [14] analyze the supply uncertainty problem for a class of EOQ models. Parlar and Perry [15] considered a stochastic inventory problem with deterministic and random yields when future supply is uncertain. Noori and Keller[3] and Gerchak[4] considered a variable-yield lot-sizing problem with stochastic demand. They showed that the solution for the backorder case is essentially an extension of the continuous-review reorder point models initiated by Hadley and Whitin[5]. Other related studies can be found in Kalro and Gohil[7], Gerchak[4], Mak [13], Noori and Keller[3], Shih[6], Parlar and Wang[9], and Wang and Gerchak[10].

In existing EOQ-type models, the yield distribution itself is assumed to be known and given. For example, the fraction distribution of defectives produced is fixed. However, to a certain extent, a firm may wish and be able to choose production processes, machines, and suppliers based on their yield distributions and associated costs. Such considerations are often mentioned in relation to modern manufacturing strategies [22]. Some recent research has attempted to improve the production process by investment in modern production technology which can impact yield distribution. Cheng[11] assumed that the unit production cost of an item increases with the yield rate. The optimal lot sizes and yield rate were obtained for this situation. Gerchak and Parlar[12] considered the problem of jointly determining the yield variance and lot sizes when the yield variability could be reduced through appropriate investment. However, they did not investigate the advantages of capital investment in reducing set-up costs. The benefits of reduced setups are well documented (eg, see Hong and Harry[13]). Recently, Lin and Hou (2005) extended the work of Noori and Keller[3] and Gerchak[4] to consider how the set-up cost and yield standard deviation can be reduced through capital investments without a limited capital budget. However, the company has recognized the resources or budget allocated are usually limited. With a limited capital budget, we would allocate these investments to reduce yield variability and set-up cost that examines the trade-offs and that allocates the investment optimally under a budget constraint. As a result, we consider an inventory system with random yield in which both the set-up cost and yield variability can be reduced through capital investment under a limited capital budget. Other several relationships between the amount of capital investment and setup cost level have been reported by many researches as in Refs. [1,3, 6-9, 11,17-18].

Based on the above arguments, this article attempts to model the production process by which yield variability can be reduced through investments in the reductions of setup cost and of yield's standard deviation under limited capital budget. To our knowledge, no previous research has addressed such scenario. In this analysis, we assume that setup cost and yield's standard deviation are functions of capital expenditure, respectively. We show that the objective function is convex. With this convexity, an iterative solution procedure is presented to find the expected optimal results. Therefore, the optimal capital investments and ordering policies that minimize the expected total annual costs for the system are appropriately determined. Finally, numerical results are provided to illustrate the results obtained and assess the performance on cost savings by adopting capital investments under a limited capital budget.

2. Notations and Assumptions

The mathematical model in this paper is developed on the basis of the following notations and assumptions.

Notations.

D = Demand rate in units/per year.

Q = the quantity ordered, in units.

Y_Q = the quantity received given that Q units are ordered, a random variable.

A_0 = the original setup cost.

σ_0 = the original yield's standard deviation.

A = the nominal setup cost per setup, a function of θ_s, with $A_0 = A(0)$.

σ = the nominal yield standard deviation, a function of θ_σ, with $\sigma_0 = \sigma(0)$.

θ_A = the capital investment required to reduce setup cost from A_0 to A.

θ_σ = the capital investment in yield's standard deviation reduction.

i = cost of capital, in \$/\$/year.

$E(Y_Q) = \mu Q$, the expected value of Y_Q given that Q units are ordered.

$\sigma_{Y_Q} = \sigma_0 Q$, the standard deviation of Y_Q given that Q units are ordered, is proportional to Q.

μ = the bias factor and $\mu = \dfrac{E(Y_Q)}{Q}$, represents the expected amount received as a proportion of the amount ordered.

h = inventory holding cost per item per year.

θ_A^* = the optimal capital investment in setup cost reduction.

θ_σ^* = the optimal capital investment in yield's standard deviation reduction.

Q^* = the optimal order quantity.

Assumptions.

1. Demand is constant and deterministic.
2. The unit variable cost is independent of the quantity order.
3. The lead time is zero and independent of the quantity ordered.
4. The quantity received is a random variable depending upon the quantity ordered.
5. $A(\theta_A)$ and $\sigma(\theta_\sigma)$ are continuously differentiable decreasing and convex in capital investment θ_A and θ_σ, respectively.
6. For the sake of generality, we assume that $\mu > \sigma_0 \ge 0$.

3. Modeling and Analysis

Based on the above notations and assumptions, we have the expected total annual costs are

$$TAC(Q) = \frac{DA_0}{\mu Q} + \frac{hQ}{2\mu}\left[\sigma_0{}^2 + \mu^2\right] \tag{1}$$

As it takes investment to reduce setup cost and yield's standard deviation, we should include an amortized investment cost in our proposed model. Therefore, the expected total annual costs of the system, $TAC_1(Q,\theta_A,\theta_\sigma)$, are composed of equation (1) and the amortized total capital costs, $i(\theta_A+\theta_\sigma)$, as follow

$$TAC_1(Q,\theta_A,\theta_\sigma) = \frac{DA(\theta_A)}{\mu Q} + \frac{hQ}{2\mu}\left[(\sigma(\theta_\sigma))^2 + \mu^2\right] + i(\theta_A+\theta_\sigma) \tag{2}$$

Notice that the TAC_1 in equation (2) is convex in Q, thus, taking the first derivative of the TAC_1 with respect to Q and equating it to zero yields the optimal lot size, Q^*, as a function of θ_A,θ_σ :

$$Q^*(\theta_A,\theta_\sigma) = \sqrt{\frac{2DA(\theta_A)}{h\left[(\sigma(\theta_\sigma))^2 + \mu^2\right]}} \tag{3}$$

Substituting equation (3) into equation (2) yields the following expression of the corresponding expected total annual costs TC:

$$TC(\theta_A,\theta_\sigma) = \sqrt{2DA(\theta_A)h\left[\frac{(\sigma(\theta_\sigma))^2}{\mu^2} + 1\right]} + i(\theta_A+\theta_\sigma) \tag{4}$$

In addition, to show the convexity of equation (4), we need following condition

$$A''(\theta_A) \ge \left[\frac{\sigma_0{}^2 + \mu^2}{\mu^2}\right]\frac{[A'(\theta_A)]^2}{2A(\theta_A)} \tag{5}$$

Theorem 1. If equation (5) holds, then $TC(\theta_A,\theta_\sigma)$ is convex with respect to θ_A and θ_σ.

Theorem 2. If equation (5) holds, then the optimal joint investments θ_A^* and θ_σ^* must satisfy

$$A'(\theta_A^*)\sigma'(\theta_\sigma^*) = \frac{\mu^2 i^2}{\sigma(\theta_\sigma^*)Dh} \tag{6}$$

and

$$2A(\theta_A^*)[\sigma(\theta_\sigma^*)\sigma'(\theta_\sigma^*)] = A'(\theta_A^*)[(\sigma(\theta_\sigma^*))^2 + \mu^2] \tag{7}$$

where $A'(\theta_A^*) = \frac{dA(\theta_A)}{d\theta_A}\Big|_{\theta_A=\theta_A^*}$ and $\sigma'(\theta_\sigma^*) = \frac{d\sigma(\theta_\sigma)}{d\theta_\sigma}\Big|_{\theta_\sigma=\theta_\sigma^*}$

4. Illustrative example

To illustrate the solution procedure described in the previous section, we consider Porteus [18] logarithmic investment cost functions to model each of setup cost reduction and yield variability reduction, described by

$$\theta_A(A) = A_s - B_s \ln(A) \qquad \text{for } 0 < A \le A_0 \tag{10}$$

and

$$\theta_\sigma(\sigma) = A_\sigma - B_\sigma \ln(\sigma) \qquad \text{for } 0 < \sigma \le \sigma_0 \tag{11}$$

where $\theta_A(A)$ and $\theta_\sigma(\sigma)$ are the investments required to achieve the setup cost A and the yield's standard deviation σ, respectively. As well as, A_s, B_s, A_σ and B_σ are positive constants and provided $2B_s > B_\sigma$. These above logarithmic cost functions can be rewritten as functions of investment as follows:

$$A(\theta_A) = \exp\left(\frac{A_s - \theta_A}{B_s}\right) \tag{12}$$

$$\sigma(\theta_\sigma) = \exp\left(\frac{A_\sigma - \theta_\sigma}{B_\sigma}\right) \tag{13}$$

According equations (12) and (13), we show that the $TC(\theta_A, \theta_\sigma)$ in equation (4) is convex with respect to θ_A and θ_σ. Hence, TC has a minimum value which is a global minimum. So, we can use equations (6) and (7), respectively, to obtain the optimal capital investment in setup cost reduction (θ_A^*) and the optimal capital investment in yield's standard deviation reduction (θ_σ^*), as follows:

$$\theta_A^* = A_s - B_s \ln\left(\frac{i^2}{Dh}[2(B_s)^2 - B_s B_\sigma]\right) \tag{14}$$

and

$$\theta_\sigma^* = A_\sigma - B_\sigma \ln\left[\sqrt{\frac{\mu^2 B_\sigma}{2B_s - B_\sigma}}\right] \tag{15}$$

Example. D = 1000, i = 0.15, A_0 = 100, σ_0 = 1.2, μ = 2.0, h = 13.25, $\theta_s(A) = A_s - B_s \ln(A)$, $\theta_\sigma(\sigma) = A_\sigma - B_\sigma \ln(\sigma)$, A_s = 8740.61, B_s = 1898, A_σ = 34.64, B_σ = 190.

In Table 1, we provide results yielded by the solution procedure proposed in this paper. In order to compare, we present the solutions for four models in Table 1: Model 1, with no investments in setup cost reduction and yield's standard deviation reduction; Mode 2, with investment in setup cost reduction alone; Model 3, with investment in yield's standard deviation reduction alone; Model 4, with optimal joint investments. From Table 1, we see that joint investments can be more efficient than the investment in one option alone, in terms of the total amount of investment and the total annual costs of the system, TC. For example, Model 2 require θ_A = \$4571.118 as the optimal investment in setup cost reduction alone and yields Q^* = 15.80 and TC = \$1255.068; Model 3 require θ_σ = \$286.398 as the optimal investment in yield standard deviation reduction alone and yields Q^* = 60.89 and TC = \$1685.154, whereas Model 4 requires θ_A = \$4084.972 and θ_σ = \$182.558 as the optimal joint investments and yields Q^* = 20.41 and TC = \$1209.530. Further, Model 2, Model 3, and Model 4 respectively have 33.9% , 11.2% , and 36.3% percentage savings as compared with Model 1.

To see the efforts of the capital investment allocated, given the total capital budget, θ_T, and report in Table 2 the solutions for Model 4. We found the θ_σ^* remains invariant as long as θ_T is greater than 182.558, which this budget is called the threshold budget (is denoted θ_{th}). In other words, we find that managers must be invested in yield standard deviation reduction before they undertake a joint investment in both.

Table1

Numerical comparison among models

	Model 1	Model 2	Model 3	Model 4
Q^*	52.68	15.80	60.89	20.41
θ_A^* ($A(\theta_A^*)$)	0 ($A(0)$=100)	4571.128 ($A(\theta_A^*)$=8.996)	0	4084.972 ($A(\theta_A^*)$=11.622)
θ_σ^* ($\sigma(\theta_\sigma^*)$)	0 ($\sigma(0)$=1.2)	0	286.398 ($\sigma(\theta_\sigma^*)$=0.266)	182.558 ($\sigma(\theta_\sigma^*)$=0.459)

TC	1898.416	1255.068	1685.154	1209.530
%*TC* sv.	--	33.9	11.2	36.3

Note: %*TC* sv. is defined the percentage of savings in *TC* as compared with Model 1.

Table 2

Optimal joint investments for a given budget

θ_T	4267.530	3000	2000	1000	500	150
Q^*	20.41	28.50	37.09	48.27	55.07	59.27
θ_A^*	4084.972	2817.442	1817.442	817.442	317.442	0
θ_σ^*	182.558	182.558	182.558	182.558	182.558	150
TC	1209.530	1245.122	1334.766	1496.637	1611.225	1709.717
%*TC* sv.	36.3	34.4	29.7	21.2	15.1	9.9

5. Concludes

When the budget is limited, appropriate investments in setup cost and yield variability reduction are an important strategy in manufacturing. In this paper, we developed ordering policies when the setup cost and yield variability could be reduced through capital investments. To explore these policies, the expected total annual costs with capital investments was formulated. We showed that the cost function is convex and developed a solution procedure to determine the optimal lot size and investment decisions. Finally, a numerical example was given to illustrate the procedure and evaluate the effects of utilizing capital investments. We find that the optimal investment in the reduction of yield standard deviation remains invariant as long as capital expenditure is greater than threshold budget. It should be emphasized that these results show that the performance on cost savings can be achieved by adopting capital investments. This approach is consistent with the JIT manufacturing philosophy, which calls for reducing setup cost and yield variability to justify smaller lot sizes with better quality.

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