Analytical and Nomogram Based Graphical Approach of Process Capability Analysis – A Comparative Study

P Nagesh[1] and Dr M S Prabhuswamy[2]
Centre for Management Studies[1],Department of Mechanical Engineering[2]
Sri Jayachamarajendra College of Engineering,
Mysore – 570 006 , Karnataka, India.
E-mail. pnagesh1973@rediffmail.com, pnagesh1973@yahoo.com, msp_sjce@yahoo.com

Abstract

The statistical techniques can be used to quantify the variability of the manufacturing process. The process capability analysis helps in quantifying the process variability and assists manufacturing by reducing the variability. Process capability of a manufacturing process can be assessed through the computations of various process capability ratios and indices. Graphical approach can be and extended to compute capability ratios. A graphical approach of process capability study is a presentation of process capability parameters in the form of nomograms. Thus graphical nomograms give quick information with fewer computations. Nomograms are developed as a graphical aid for process capability analysis and thus it can be used at shop floor levels for the assessment of variability of the manufacturing process. This paper contains an extensive discussion on quantifying the variability of the manufacturing process and analyzing the process capability in both analytical and graphical methods as a comparative study.

Keywords: Process capability analysis, Graphical approach, Nomograms.

Introduction

The main objectives of process management are to prevent the defects and to minimize the variability in manufacturing process. To reduce the variability it needs to establish the relationship between process variable and product results. Even though product meets the specification limits and the process is in the state of control, the actions may be initiated to quantify and further reducing the process variability to gain marvelous advantages [1]. The statistical techniques can be used to quantify the variability of the manufacturing process. The process capability analysis helps in quantifying the process variability and assists manufacturing by eliminating or greatly reducing the variability.

Methodology

Methodology involves following steps.

1. The relationship between various components of the process capability analysis is established.
2. The nomograms are developed to the above relationships.
3. Analytical method is extended to graphical approach.
4. A case study is evaluated and results of analytical and graphical approach are compared.

1 Process Capability Analysis

Process capability is inherent variation of the product turned out by the process. Process capability provides the quantified prediction of the process adequacy and refers to the uniformity of the process. It is a measurement with respect to the inherent precision of a manufacturing process. Process capability analysis is a measurement of quality performance capability of the process with given factors and under statistical control conditions. Estimate of process capability may be in the form of probability distribution having a specified shape, center and spread. In the sense, a process capability analysis may be performed without regard to specifications on the quality characteristic. The process capability can be measured and analyzed through the computations of various process capability indices by analytical methods. Graphical approach can be used to measure the process capability. The graphical approach of process capability study makes it easier to assess the variability of the manufacturing process with less computation and time.

3 Case Study

To analyze the variability of the manufacturing process, the driver gear boring operation in an automotive industry is chosen and twenty samples containing five sub-samples is selected. The observations of boring quality characteristics are noted down as indicated in the Table – 4 below.

Machine: ACE H -63 CNC Machining center.

Part Name: Drive gear. Operation: Boring Specifications: 210.745 mm to 210.795 mm

(Source: Results obtained by authors during process capability studies)

No	1	2	3	4	5	$\overline{X}$	R
1	210.795	210.790	210.790	210.795	210.795	210.793	0.005
2	210.785	210.785	210.775	210.775	210.775	210.779	0.010
3	210.770	210.775	210.770	210.775	210.770	210.772	0.005
4	210.770	210.765	210.770	210.775	210.770	210.770	0.01
5	210.765	210.765	210.765	210.760	210.760	210.763	0.005
6	210.760	210.755	210.785	210.790	210.785	210.775	0.035
7	210.780	210.775	210.775	210.770	210.765	210.773	0.015
8	210.765	210.765	210.765	210.775	210.770	210.768	0.01
9	210.775	210.770	210.770	210.770	210.770	210.771	0.005
10	210.765	210.775	210.775	210.775	210.770	210.772	0.01
11	210.770	210.775	210.770	210.765	210.760	210.768	0.015
12	210.785	210.785	210.780	210.780	210.780	210.782	0.005
13	210.775	210.775	210.775	210.780	210.780	210.777	0.005
14	210.775	210.770	210.765	210.765	210.755	210.766	0.02
15	210.795	210.795	210.785	210.780	210.775	210.786	0.02
16	210.775	210.765	210.760	210.760	210.755	210.763	0.02
17	210.770	210.775	210.770	210.775	210.770	210.772	0.005
18	210.770	210.765	210.765	210.765	210.760	210.765	0.01
19	210.790	210.790	210.790	210.790	210.780	210.788	0.01
20	210.770	210.765	210.765	210.765	210.765	210.766	0.005

4 COMPUTATIONS AND ANALYSIS BY ANALYTICAL METHOS

4.1 Process Capability Ratio-Cp

Measured quality characteristics of parts are tabulated. Sample 1 to 20 is indicated in first column of Table – 4 and each row indicates five sub-samples. $\overline{X}$ is the mean value of sub-samples and R is the range, the difference between highest and lowest value of observations among each sub-sample.

The means of sample means ($\overline{\overline{X}} = \mu$) and average range($\overline{R}$) value is computed. The process standard deviation is the ratio of average range to d_2, a statistical constant which can be obtained from the factor $d_2 = \overline{R} / \sigma$ column in a statistical table [16].

$\sigma = 0.01125 / 2.326 = 0.0048$.

$C_p = (USL – LSL) / 6 \sigma = (210.795 – 210.745) / (6 \times 0.00483) = 1.736$.

This value of C_p implies that natural tolerance limits in the process are well inside the specification limits. From the table-2, for PCR = 1.736, it is noticed that the probability of 0.34 ppm defectives are there in manufacturing process.

4.2 Percentage of Specification Band Used By the Process:

$C_r = (1/ C_p) \times 100 =(1/ 1.725)\times 100 =57.6\%$

This means that the manufacture process has a wide tolerance bandwidth while it uses 57.6% of specification band.

4.3 PCR-C_{pk} : The grand mean of the case analysis is 210.773.

$$C_{pk} = \text{Min}\left\{ C_{pu} = \frac{(210.795 - 210.773)}{3 \times 0.0048},\ C_{pl} = \frac{(210.773 - 210.745)}{3 \times 0.0048} \right\}$$

C_{pk} = Minimum { C_{pu} = 1.527 , C_{pl} = 1.944 } .C_{pk} = 1.527.

It should be noted that C_p measures potential capability of the manufacturing process, whereas C_{pk} measures actual capability

4.4 Estimation of Process Centering

a) Estimation of C_{pm} :

$$C_{pm} = \frac{C_p}{\sqrt{1+\xi^2}} = \frac{1.736}{\sqrt{1+(0.625)^2}} = 1.472$$

Where, $\xi = (\mu - T)/\sigma$

= (210.773–210.77) / 0.0048 = 0.625 and

T = ½ (USL+LSL) = ½ (210.795 + 210.745) = 210.770 mm.

b) Estimation of C_{pkm} :

$$C_{pkm} = \frac{C_{pk}}{\sqrt{1+\xi^2}} = \frac{1.518}{\sqrt{1+(0.625)^2}} = 1.294$$

The case analysis reveals that, C_{pk} = 1.527 , C_{pm} = 1.472 and C_{pkm} = 1.294 that implies C_{pk} C_{pm} and C_{pkm} are not very nearer in their magnitude and hence the process under study is not exactly centered.

4.5 Confidence Intervals &Tests on PCR

a) Confidence intervals & tests on PCR -C_p

$$C_p \sqrt{\frac{X^2_{(1-\alpha/2),\, n-1}}{n-1}} \le C_p \le C_p \sqrt{\frac{X^2_{\alpha/2,\, n-1}}{n-1}}$$

$$1.736 \sqrt{\frac{74.22}{99}} \le C_p \le 1.736 \sqrt{\frac{129.56}{99}}$$

$$1.503 \le C_p \le 1.981$$

4.6 b) Estimation of confidence interval for C_{pk}

$$C_{pk}\left[1 - Z_{\alpha/2}\sqrt{1/(9\,n\,C_{pk}^{\,2}) + 1/2(n-1)}\right] \le C_{pk} \le C_{pk}\left[1 + Z_{\alpha/2}\sqrt{1/(9\,n\,C_{pk}^{\,2}) + 1/2(n-1)}\right]$$

$$1.527\left[1 - 1.96\sqrt{1/(9 \times 100 \times (1.527)^2) + 1/(2 \times 99)}\right] \le C_{pk} \le 1.527\left[1 + 1.96\sqrt{1/(9 \times 100 \times (1.527)^2) + 1/(2 \times 99)}\right]$$

$$1.304 \le C_{pk} \le 1.749$$

Based on the confidence intervals and tests on PCR- C_p, reveals that C_p value is $1.503 \le C_p \le 1.981$ and estimation of confidence interval for C_{pk} is $1.304 \le C_{pk} \le 1.749$. However, for these values test of hypothesis can be carried out.

4.7 Testing and Hypothesis:

Let us $H_o = C_p = 1.736$, $H_1 = C_p < 1.736$ with $\alpha = \beta = 0.10$

C_p (High) / C_p (Low) = 1.981/1.503= 1.32. From table-2, the ratio 1.32 corresponds to n=50. C = 1.13 x C_p (low) = 1.13 x 1.503 = 1.67. This means that, to demonstrate capability the supplier or process must take a sample of n= 50 parts and the sample PCR-C_p must exceed 1.687.The case analysis reveals that PCR-C_p is 1.736, which is greater than 1.687 and hence the process under study is capable.

4.8 Estimation of Economic Loss

Using this cost as a reference value, a loss function can be constructed as shown in figure -1. Since, this cost constant is not readily available at all situations; an arbitrary assumed value of Rs 10,000 is considered for the case analysis. Economic loss due to process variability is given by

$$L= C\{[(USL - LSL)^2 /4][(1/ 3\, C_p)^2 + K^2)]\}$$

$$K = \frac{2\,|\mu - T|}{(USL - LSL)} = \frac{2\,(210.773 - 210.770)}{(0.050)} = 0.12$$

$$L=C\{[(USL - LSL)^2/ 4][(1/ 3\, C_p)^2+K^2)]\}=\{ [(0.05)^2 / 4] [1 / (3 \times 1.736)^2+0.12^2]\}$$

L/C = 0.000032.

Where, C= Cost constant, USL and LSL are upper and lower specification limits, C_p = process capability ratio and K is scaled distant factor which represents shift in target. Say, for C= cost constant of Rs 10000, and then the economic loss will be Rs 0.32. This indicates financial loss due to variability associated with the boring operation in case analysis.

5 THE NOMOGRAM BASED PROPOSED GRAPHICAL APPROACH

In order to make the process capability analysis more simple and understandable at the shop floor level graphical aids are appropriate. Usage of nomogram reduces the computations [17]. This also helps to create the awareness and quality concepts at the shop floor level.

5.1 Relationship between Components of Process Capability Ratios

5.11 The process capability ratio- C_p

C_p gives quick observation to determine whether the process is capable of meeting specification. The relationship between natural variation and specification is quantified by a measure called process capability ratio- C_p.

Process capability ratio is $C_p = \frac{(USL - LSL)}{6\sigma}$ ....(1)

Where, USL = Upper specification limit, LSL = Lower specification limit and σ = Process standard deviation. It can be noted from equation (1) that Process capability ratio is C_p is directly proportional to tolerance and inversely proportional to process standard deviation.

5.12 Percentage of Specification Band Used By the Process: The PCR can be indicated as C_r index, which measures the percentage of tolerance used by process. It is the ratio of process spread to tolerance spread. $C_r = (1/ C_p) \times 100$

5.13 The process capability index - C_{pk}

C_{pk} considers process average into account and evaluates half the process spread with respect to where the process is actually located. C_{pk} is calculated with an equation $C_{pk} =$ Minimum$\{C_{pu}, C_{pl}\}$.

Generally, if $C_p = C_{pk}$ the process is centered at mid point of specifications and if $C_{pk} < C_p$ the process is off-center. The magnitude of C_{pk} relative to C_p is a direct measurement of how off-center the process is operating.

5.14 Scaled distant factor (K)

$$K = \frac{2\,|\,\mu - T\,|}{(USL - LSL)}$$

Scaled distant factor represents shift in target

5.15 Relationship between C_{pk}, C_p and K

It is established through an equation $C_{pk} = C_p\ (1\text{-}K)$. For the given value of K, the relationship between C_p and C_{pk} is linear and as the C_p increases C_{pk} also increases.

Above relationships are used to develop the nomograms for the process capability analysis.

6 NOMOGRAMS

Nomograms are drawings made in order to replace analytical computations and thus save the time [18]. The graphical approach of the process capability analysis requires the development of nomograms. The process capability parameters and their relationships are drawn in the form of nomograms as shown in the figure 1 and 2.Figure-1 establishes the relationship between tolerance (Δ) of the product, scaled distant factor and the difference between processes mean and target value. Difference between process mean (X) and process target (T) is noted as X. Nomogram (figure-1) can be used to determine scaled distant factor K for the given tolerance (Δ) and X.

Figure-2 establishes the relationship between process standard deviation, process capability ratio- C_p and The process capability index - C_{pk} .The standard deviation of the process (σ) is used to find process capability ratio (PCR)- C_p and hence the percentage of the specification band used by the process is obtained. K value obtained through Figure-1 is used to get the process capability index C_{pK}.

6.1 Illustration

The case evaluation reveals that, the process mean ($\bar{X}$) = 210.773 mm , Process standard deviation (σ) = 0.0048, Process Target = 210.770 mm. The difference between processes mean and target value (X) = (210.773 - 210.770) = 0.003 mm and Tolerance of the product = 0.050 mm.

In figure-1 the tolerance value 0.050 mm is chosen in the horizontal axis and moved upwards till it get intersected at X = 0.003. The intersection point is projected leftwards to get scaled distant factor K, which is equal to 0.13.

In figure-2, Process standard deviation 0.0048 is chosen in the horizontal the horizontal axis and moved upwards till it get intersected at tolerance (Δ)= 0.050. The intersection point is projected leftwards to get process capability ratio (PCR)- C_p = 1.75 and continued the projection horizontally to get the percentage of the specification band used by the process which is equal to 57.1 %. (1-K) is calculated as (1-0.13) = 0.87. By connecting (1-K) and Cp, the value of C_{pK} is obtained which is equal to 1.5. Further computations follow usual procedure.

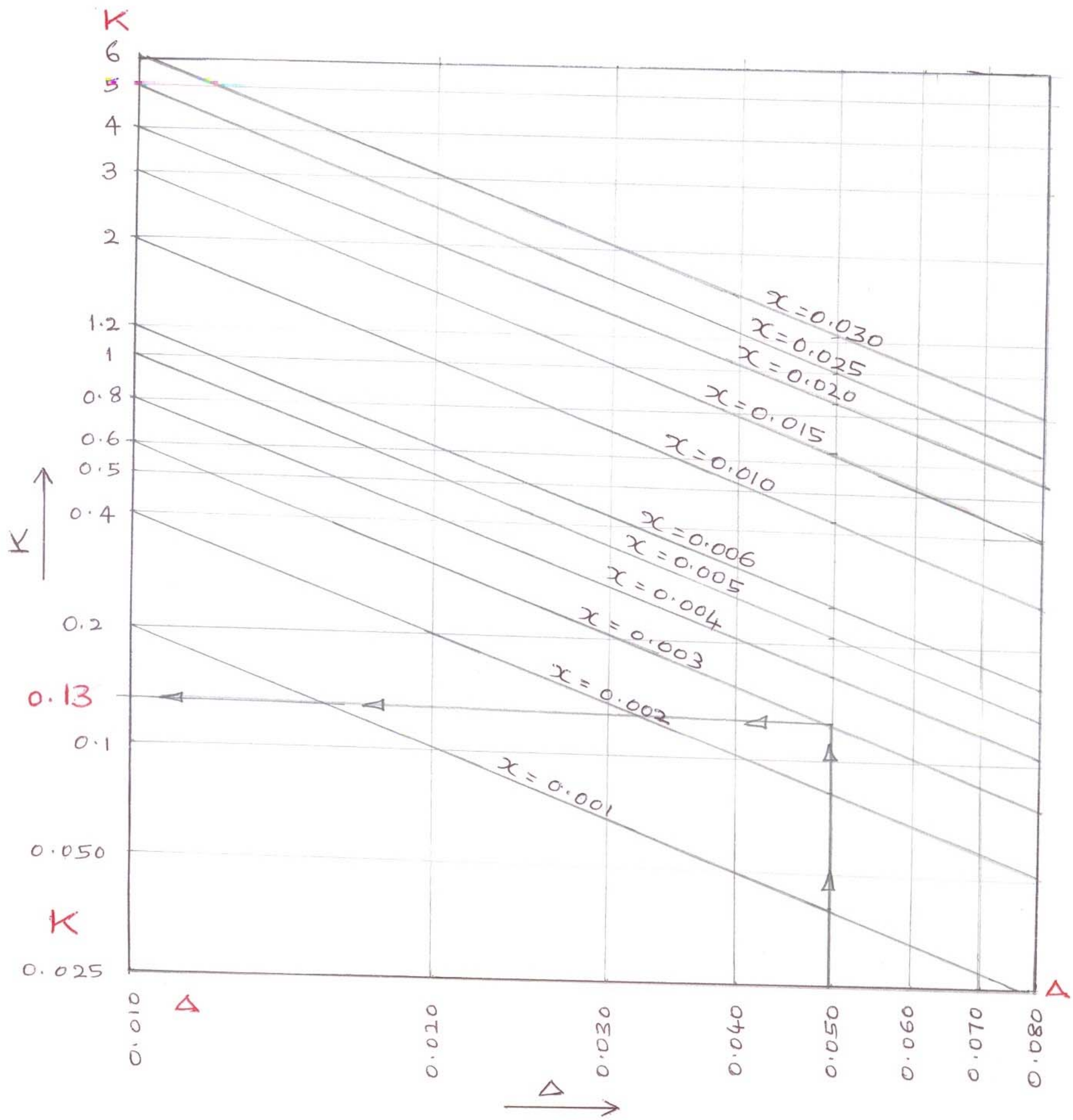

Figure –1, Nomogram

Input to the nomogram - Tolerance and X = (Process Mean – Process Target)

Out put – Scaled Distant Factor (K)

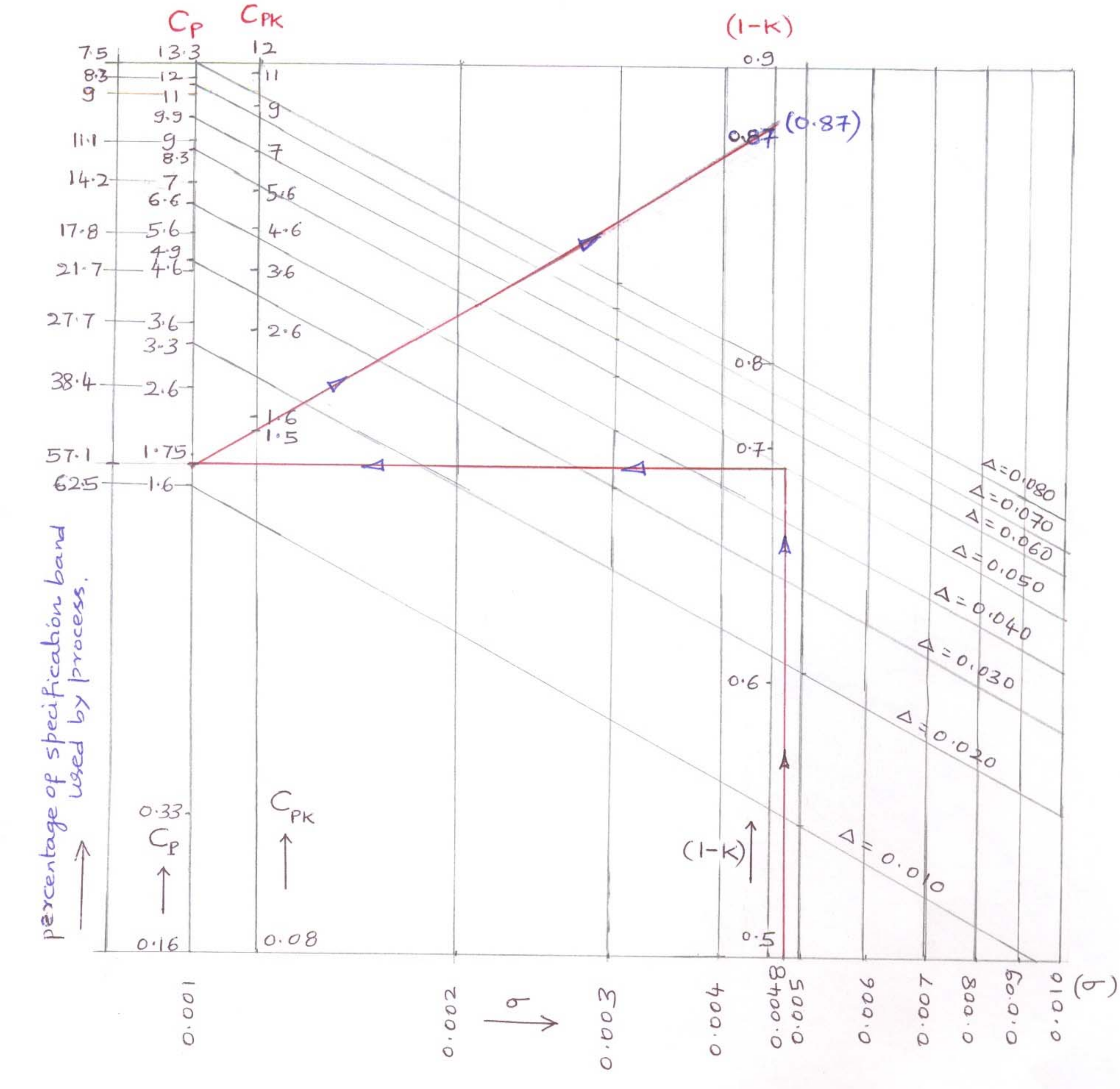

Figure –2, Nomogram

Input to the nomogram - Process Standard Deviation, Tolerance and Scaled Distant Factor –K

Out put - Process Capability ratio – Cp, Percentage of specification used by the process and Cpk.

7 Computations from the proposed model

7.1 Process standard deviation

The process standard deviation is the ratio of average range to d_2, a statistical constant which can be obtained from the factor $d_2 = \bar{R} / \sigma$ column in a statistical table [Reference -20, pp717].With the help of statistical table the value of d_2 for a sub-sample size of five is noted as 2.326 and the process standard deviation is estimated by $\sigma = \bar{R}/ d_2$

Process Standard Deviation = Average Range / d_2 = 0.01125/2.326 = 0.0048

7.2 Process Capability Ratio – C_p

Process capability ratio - C_p measures the ability of process to manufacture product that meets specification. Which is a ratio of tolerance to six times the process standard deviation.

From the nomogram, Process Capability Ratio – C_p is obtained as 1.75.

This value of C_p implies that natural tolerance limits in the process are well inside the specification limits. From the table-2, for PCR- C_p = 1.75, it is noticed that the probability of 0.34 ppm defectives are there in manufacturing process.

7.3 PCR – C_{pk} :

From the nomogram, Process Capability Ratio – C_{pk} is obtained as = 1.5

It should be noted that C_p measures potential capability of the manufacturing process, whereas C_{pk} measures actual capability

7.4 Estimation of C_{pm} :

C_{pk} does not tell us about location of mean in the interval from USL and LSL. The way to address this difficulty is to use a PCR - C_{pm} that is better indicator of centering.

$$C_{pm} = \frac{C_p}{\sqrt{1+\xi^2}} = \frac{1.75}{\sqrt{1+(0.625)^2}} = 1.48$$

Where, $\xi = = \bar{X} / \sigma$ =0.003 / 0.004= 0.625

T = ½ (USL+LSL) = ½ (210.795 + 210.745) = 210.770 mm

7.5 Estimation of C_{pkm} :

PCR – C_{pkm} Equation has increased sensitivity to departures of process mean μ from designed target T.

$$C_{pkm} = \frac{C_{pk}}{\sqrt{1+((\mu - T)/\sigma)^2}} = \frac{C_{pk}}{\sqrt{1+\xi^2}}$$

This is also called “Third generation” PCR.

$$C_{pkm} = \frac{C_{pk}}{\sqrt{1+\xi^2}} = \frac{1.5}{\sqrt{1+(0.625)^2}} = 1.27$$

7.6 Estimation of Economic Loss.

Economic loss due to process variability is given by $L= C\{[(USL - LSL)^2 /4][(1/ 3\, C_p)^2 + K^2)]\}$

Where, C= Cost constant, USL and LSL are upper and lower specification limits, C_p = process capability ratio and K is scaled distant factor which represents shift in target and is given by **From nomogram the value of K is obtained as 0.13**

$L=C\{[(USL - LSL)^2/ 4][(1/ 3 C_p)^2+K^2)]\}$

$L/C =\{ [(0.05)^2 / 4] [1 / (3 \times 1.75)^2+0.13^2]\} = 0.000033.$

8 COMPARISON OF RESULTS

Parameter	Analytical Method	Graphical Approach
Process Capability –PCR-C_p	1.73	1.75
Process capability ratio - C_{pk}	1.52	1.50
Process capability ratio – C_{pm}	1.47	1.48
Process capability ratio – C_{pkm}	1.29	1.27
Scaled Distant Factor -K	0.12	0.13
Loss function value	0.000032	0.000033

The results of process capability study of the given manufacturing process reveals that, Nomogram Based Graphical Approach values are very nearer in their magnitude as compared to analytical method and hence graphical approach could be used in place of analytical method.

9 ADVANTAGES OF PROPOSED GRAPHICAL APPROACH.

1. The proposed Nomogram Based Graphical Approach is simple and pictorial representation and can be easily understand.
2. The approach can be used as a graphical aid at shop floor.
3. Proposed approach reduces the large amount of the computations and thus saves time.
4. The variability of the manufacturing process can be studied and quantified easily and this approach can create the awareness of process management at the shop floor level

10 LIMITATIONS OF THE APPROACH

The process mean and process standard deviation is to be computed from analytical method.

11 CONCLUSION

The results of process capability study of the given manufacturing process reveals that, graphical values of parameters approaches very nearer to the magnitude of the analytical values and hence graphical approach could be treated as equivalent to analytical method. Graphical approach can be used to study the variability of manufacturing process. Graphical approach of the process capability studies gives quick information with fewer computations. It is one of the tools to convey the results through which it is easy to make inference on the data. The approach helps a worker in the shop floor can make the assessment about the process parameters. Thus it also helps to process management and identifies opportunities for improving quality and operational performance.

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